Stability of Plasmas Held by Radiation Pressure

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An extension of a simplified model of Brillouin scattering instability to arbitrarily inhomogeneous media is presented. It is shown that light pressure, if treated self-consistently, always drives the plasma unstable.

Most of the studies of stability of plasmas in the presence of electromagnetic waves are concerned with wave decay or parametric effects, such as Brillouin scattering, in homogeneous or slightly inhomogeneous plasmas [1]. In contrast this short contribution is devoted to the stability investigation of arbitrarily inhomogeneous plasmas held by light pressure, the importance of which was pointed out years ago [2]. The Brillouin type instability is due to the effect that a density perturbation of twice the local light wavelength modifies the electromagnetic wave in such a way that its radiation pressure tends to increase the original perturbation [3]. Since the frequency shift of the scattered electromagnetic wave is small ($\omega_{\text{acoustic}} \ll \omega_{\text{light}}$), it is a good approximation to use the same index of refraction for the incident and scattered waves, at least far from the critical density. The essential features of possible instabilities are thus preserved if a simplified response of the plasma to the light as described in the following equations [4] is assumed:

$$\varrho\left(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x}\right) = -\,s^2\,\frac{\partial \varrho}{\partial x} - \mu\,\varrho\,\frac{\partial}{\partial x}\langle E E^*\rangle\,, \tag{1}$$

$$\frac{\partial}{\partial t}\,\varrho + \frac{\partial}{\partial x}\,(\varrho\,u) = 0\,, \tag{2}$$

$$\partial^2 E/\partial x^2 + \varkappa_0^2\,n^2\,E = 0\,, \tag{3}$$

where $n^2 = 1 - \varrho(x)/\varrho_c$. The symbols have the usual meaning [4] (u is the x component of velocity, ϱ_c is the critical density, E is the electric field, $\varkappa_0$ is the vacuum wave number, s is the sound velocity). The equations are valid if the fluid is 1-dimensional and isothermal, behaves hydrodynamically, and dissipation can be neglected.

We are interested in a time evolution of u which is much longer than the period of the light wave, and so it is justified to replace the averages on the rapid time scale in Eq. (1) by the x dependent amplitudes $E(x)$. Furthermore, $\varkappa_0^2 n^2$ is real and allows us to consider only real $E(x)$ so that we replace $\langle E E^*\rangle$ by E^2.

If we assume a static equilibrium for the zero order, Eqs. (1) to (3) reduce to

$$s^2\,\frac{\partial}{\partial x}\,\varrho_0 + \mu\,\varrho_0\,\frac{\partial E_0^2}{\partial x} = 0\,, \tag{4}$$

$$\frac{\partial^2 E_0}{\partial x^2} + \varkappa_0^2\left(1 - \frac{\varrho_0(x)}{\varrho_c}\right) E_0 = 0\,, \quad \varrho_0 < \varrho_c\,. \tag{5}$$

The linearized equations around ϱ_0, E_0 are then

$$\varrho_0\,\frac{\partial u}{\partial t} = -\,s^2\,\frac{\partial \varrho_1}{\partial x} - \mu\,\varrho_1\,\frac{\partial E_0^2}{\partial x} - 2\,\mu\,\varrho_0\,\frac{\partial}{\partial x}\,(E_0\,E_1)\,, \tag{6}$$

$$\frac{\partial}{\partial t}\,\varrho_1 + \frac{\partial}{\partial x}\,(\varrho_0\,u) = 0\,, \tag{7}$$

$$\frac{\partial^2 E_1}{\partial x^2} + \varkappa_0^2\left(1 - \frac{\varrho_0}{\varrho_c}\right) E_1 = \varkappa_0^2\,E_0\,\frac{\varrho_1}{\varrho_c}. \tag{8}$$

E_1 can be expressed in terms of Green's function $G(x, x')$

$$E_1 = \varkappa_0^2 \int_a^b G(x, x')\,E_0(x')\,\frac{\varrho_1(x')}{\varrho_c}\,\mathrm{d}x'\,. \tag{9}$$

By taking the time derivative of Eq. (6) and using Eq. (9) the system (6)–(8) reduces to

$$\varrho_0\,\frac{\partial^2 u}{\partial t^2} = s^2\,\frac{\partial^2}{\partial x^2}\,(\varrho_0\,u) + \mu\,\frac{\partial (E_0^2)}{\partial x}\,\frac{\partial}{\partial x}\,(\varrho_0\,u) + 2\,\mu\,\varkappa_0^2\,\frac{\varrho_0}{\varrho_c}\,\frac{\partial}{\partial x}\left[E_0 \int_a^b G(x, x')\,E_0(x')\,\frac{\partial}{\partial x'}\,(\varrho_0\,u)\,\mathrm{d}x'\right] \tag{10}$$

or

$$\varrho_0\,\ddot{u} + F\,u = 0\,, \tag{11}$$

where F is the integro-differential operator on u of the R.H.S. of Equation (10). The reader can easily check that the differential part of F is symmetric for vanishing u at the boundaries. The integral operator is also symmetric because Green's function inverts the symmetric operator on the L.H.S. of

Eq. (8) and then has to be symmetric with respect to interchange of x and x'. This property of Eq. (11) allows a necessary and sufficient condition of stability to be derived in the form of an energy principle as known from Ref. [5].

Let $\delta W = \int_a^b u\, F\, u\, \mathrm{d}x$, i.e.

$$\delta W = s^2 \int_a^b [(\varrho_0\, u)_x]^2 \frac{\mathrm{d}x}{\varrho_0} + 2\,\frac{2\,\mu\,\varkappa_0^2}{\varrho_c} \int_a^b \int_a^b G(x, x')\, E_0(x')\, (\varrho_0\, u)_{x'}\, E_0(x)\, (\varrho_0\, \mu)_x\, \mathrm{d}x'\, \mathrm{d}x\,. \tag{12}$$

If $\delta W > 0$ for all u vanishing at $x = a$ and $x = b$, the system is stable. If for any test function u, vanishing at a and b, $\delta W < 0$ holds, the system is unstable.

This means that without the self-consistent reaction of the plasma to the light, i.e. $E_1 \equiv 0$, the equilibrium is stable. Let us investigate this self-consistent response by analyzing the properties of Green's function $G(x, x')$ of Equation (8). The operator

$$L \equiv \frac{\partial^2}{\partial x^2} + \varkappa_0^2 \left(1 - \frac{\varrho_0}{\varrho_c}\right)$$

has a null eigenvalue if

$$\varkappa_0 (b - a) \overline{(1 - \varrho_0/\varrho_c)^{1/2}} = \alpha_{\mathrm{crit}} \approx 1$$

where the bar indicates an average in x. When this condition is barely satisfied, Green's function becomes very large and can change sign. This is easy to understand from the fact that if L has the eigenvalue λ, L^{-1} has the eigenvalue λ^{-1}. So $G(x, x')$ can be negative and large if

$$\varkappa_0 (b - a) \lesssim \overline{(1 - \varrho_0/\varrho_c)^{-1/2}}\,.$$

This means that the double integral in Expression (12) can be made negative and large enough to yield $\delta W < 0$ for a properly chosen test function.

In conclusion, we can say that the plasma is unstable if the ponderomotive force is perturbed self-consistently, no matter how large the modulation of the light and the plasma inhomogeneity are.

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